Available online at www.sciencedirect.com

SciVerse ScienceDirect

Procedia Engineering 29 (2012) 1223 – 1227

Procedia Engineering

www.elsevier.com/locate/procedia

2012 International Workshop on Information and Electronics Engineering (IWIEE)

Subcell FDTD Based on Z Transforms Modeling of Electrically Thin Dispersive Layers

Chaoqun Jiao[a, b*], Zhanjun Xia[a, b]

[a]*School of Electrical Eng., Bejing Jiaotong University, No 3 Shang Yuan Cun, Haidian District, 100044,*
[b]*China State Key Laboratory of Millimeter Waves, Nanjing, 210096, China*

Abstract

A novel technique for treating electrically thin dispersive layers with the finite-difference time-domain (FDTD) method based on Z transforms is introduced. The proposed model is based on the subcell technique, where the constitutive relations are locally averaged in the FDTD grid. The most significant feature of the proposed model is its ability to model rather complicated dispersive layers having multiple pole pairs. Then, based on the model above, the model for pulse reflection from a coated ideal conductor is proposed. The models are validated with several numerical examples making comparison with the exact results.

Electrically thin Dispersive layers, Finite difference time domain (FDTD), Subcell FDTD, Z Transforms

1. Introduction

Many microwave devices contain electrically thin layers. Therefore, the numerical modeling of such structures is of interest. The finite-difference time-domain (FDTD) method has been widely accepted as an efficient tool for the accurate solving of a great variety of electromagnetic problems [1-2]. The present problem, modeling of electrically thin dispersive layers, may be solved basically in the following three ways: 1) with direct and fine enough discretization of the fields inside the layer; 2) using the surface impedance boundary conditions (SIBCs)[3-6]; 3) by locally modifying the update equations to account for the layer[7-8]. Notice that the previous subcell techniques are not applicable to dispersive layers. Mikko

* Corresponding author. Tel.: +86-010-51687108.
E-mail address: chaoqunjiao@126.com

doi:10.1016/j.proeng.2012.01.117

proposed a subcell model for general dispersive layers, which was based on auxiliary variables method in [9]. In this paper, we formulate a new subcell technique based on Z transforms [10], which allows modeling quite general dispersive layers, possibly having multiple pole pairs. The proposed model reduces to the model by Maloney and Smith in the case of dielectric and conductive layers. The new model is formulated in the general three-dimensional (3-D) case in Section II, and validation study is conducted with two-dimensional (2-D) and one- dimensional (1-D) FDTD programs in Section III. Quite good agreement with the analytical results and results from auxiliary variables method is observed.

2. Subcell Technique Based on Z Transforms for Dispersive Layers

The basic idea of the model is quite simple: we will average the electric and magnetic flux densities so that they will simulate the presence of a thin dispersive layer. The layer is assumed to be located in free space, although this need not necessarily be the case. The field components both tangential and normal to the layer will be affected by the model. Consider deriving the update equations for the tangential magnetic-field components in the vicinity of a dispersive layer of thickness *d* and with the frequency-dependent isotropic permeability

$$\mu(\omega) = \mu_0\left(\mu_\infty + \sum_{k=1}^{P} \beta_{m,k} \Big/ \left(\omega_{0m,k}^2 - \gamma_{m,k}\omega^2 + j\delta_{m,k}\omega\right)\right) \tag{1}$$

where *P* is the number of pole pairs and the subscript *k* refers to the *k* th pole pair. The subscript *m* refers to the magnetic layer. An appropriate choice of the parameters in the above expression allows us to obtain a layer of Lorentz, Debye, or Drude type as special cases. Similarly, the expression for the permittivity is taken to be of the form

$$\varepsilon(\omega) = \varepsilon_0\left(\varepsilon_\infty + \sum_{k=1}^{P} \beta_{e,k} \Big/ \left(\omega_{0e,k}^2 - \gamma_{e,k}\omega^2 + j\delta_{e,k}\omega\right)\right) \tag{2}$$

with analogous definitions of the parameters as above. Let the layer partially fill a single plane of FDTD cells in the 3-D FDTD lattice. A slice of the FDTD lattice in *xy*-plane is shown in Fig. 1.

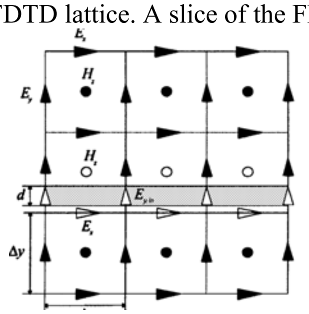


Fig.1. Slice of the FDTD lattice in the xy-plane

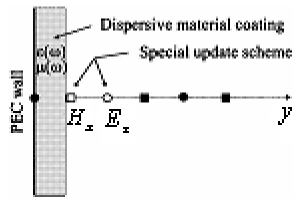


Fig.2. Problem geometry in a 1-D problem

If the volume fraction occupied by the layer is α, then we may calculate the averaged magnetic flux density inside the cells containing the dispersive layer according to $\mathrm{B} = \alpha\mu(\omega)\mathrm{H}_{\text{layer}} + (1-\alpha)\mu_0\mathrm{H}_{\text{free space}}$ with $0 \le \alpha \le 1$. Using the auxiliary term $\mathrm{S}_{\mathrm{m,k}}(\omega)$ associated with the *k*th pole pair, defined as

$$\mathrm{S}_{\mathrm{m,k}}(\omega) = \frac{\beta_{m,k}}{\omega_{0m,k}^2 - \gamma_{m,k}\omega^2 + \mathrm{j}\delta_{m,k}\omega}\mu_0\mathrm{H}(\omega) \tag{3}$$

We obtain the equation

$$\mathrm{B}(\omega) = \alpha\mu_0\mu_\infty\mathrm{H}(\omega) + \sum_{\mathbf{k}=1}^{P}\mathrm{S}_{m,k}(\omega) + (1-\alpha)\mu_0\mathrm{H}(\omega) \tag{4}$$

An alternative form of the equation (3) is

$$\mathrm{S}_{m,k}(\omega)=\frac{\theta_{m,k}\varphi_{m,k}}{\phi_{m,k}{}^{2}+\varphi_{m,k}{}^{2}+\mathrm{j}2\phi_{m,k}\omega+(j\omega)^{2}}\mu_0\mathrm{H}(\omega) \tag{5}$$

where

$$\phi_{m,k}=\frac{\delta_{m,k}}{2\gamma_{m,k}},\ \varphi_{m,k}=\frac{\sqrt{\omega_{0m,k}^{2}-\delta_{m,k}^{2}/4}}{\gamma_{m,k}},\ \theta_{m,k}=\frac{\beta_{m,k}}{\sqrt{\omega_{0m,k}^{2}-\delta_{m,k}^{2}/4}}$$

The corresponding Z transforms of the equation (5) is

$$\mathrm{S}_{m,k}(z)=\frac{e^{-\phi_{m,k}\cdot\Delta t}\cdot sin(\varphi_{m,k}\cdot\Delta t)\cdot\Delta t\cdot z^{-1}\theta_{m,k}\mu_0}{1-2e^{-\phi_{m,k}\cdot\Delta t}\cdot cos(\varphi_{m,k}\cdot\Delta t)\cdot\Delta t\cdot z^{-1}+e^{-2\phi_{m,k}\cdot\Delta t}}\mathrm{H}(z) \tag{6}$$

The corresponding Z transforms of the equation (4) is

$$\mathrm{B}(z)=\alpha\mu_0\mu_\infty\mathrm{H}(z)+\sum_{k=1}^{P}\mathrm{S}_{m,k}(z)+(1-\alpha)\mu_0\mathrm{H}(z) \tag{7}$$

The corresponding sampled time domain equation (6) is

$$\begin{aligned}\mathrm{S}_{m,k}^{n}&=2e^{-\phi_{m,k}\cdot\Delta t}\cdot cos(\varphi_{m,k}\cdot\Delta t)\cdot\Delta t\cdot\mathrm{S}_{m,k}^{n-1}-e^{-2\phi_{m,k}\cdot\Delta t}\cdot\mathrm{S}_{m,k}^{n-2}+\\&\quad e^{-\phi_{m,k}\cdot\Delta t}\cdot sin(\varphi_{m,k}\cdot\Delta t)\cdot\theta_{m,k}\cdot\Delta t\cdot\mu_0\mathrm{H}^{n-1}\end{aligned} \tag{8}$$

Therefore, the FDTD simulation for H_z becomes

$$B_{\mathbf{z}}\Big|_{\mathbf{i+1/2,j+1/2,k}}^{\mathbf{n+1/2}}=B_{\mathbf{z}}\Big|_{\mathbf{i+1/2,j+1/2,k}}^{\mathbf{n-1/2}}-\Delta t\cdot(\nabla\times E)_{\mathbf{z}}\Big|_{\mathbf{i+1/2,j+1/2,k}}^{\mathbf{n}} \tag{9}$$

$$H_z\Big|_{i,j+1/2,k+1/2}^{n+1/2}=\frac{1}{[1+\alpha(\mu_\infty-1)]\mu_0}B_z\Big|_{i+1/2,j+1/2,k}^{n+1/2}-\frac{1}{[1+\alpha(\mu_\infty-1)]\mu_0}\sum_{k=1}^{P}S_{m,k}\Bigg|_{i+1/2,j+1/2,k}^{n} \tag{10}$$

The derivation of the update equations for the electric field components in the case of a layer in free space is quite analogous and is not shown here.

However, in the case when metal is coated with the layer, we must account for the fact that the tangential electric field decays to zero in the vicinity of the ideal conductor. This situation is most conveniently described in a 1-D case. Suppose that there is an ideally conducting wall at $y=0$ as shown in Fig. 2 and let the coating on the wall have a thickness d. Using Z transform, we obtain the update equation for the electric field $E_z\big|_1$ next to the metal wall in the 1-D case

$$\begin{aligned}E_z\Big|_1^{n+1}&=\frac{2\varepsilon_{r,\infty,ave}\varepsilon_0-\sigma_{ave}\Delta t}{2\varepsilon_{r,\infty,ave}\varepsilon_0+\sigma_{ave}\Delta t}E_z\Big|_1^{n}+\frac{2\Delta t}{\Delta x(2\varepsilon_{r,\infty,ave}\varepsilon_0+\sigma_{ave}\Delta t)}\left(H_x\Big|_{1/2}^{n+1/2}-H_x\Big|_{3/2}^{n+1/2}\right)\\&-\frac{2\alpha\Delta t}{2\varepsilon_{r,\infty,ave}\varepsilon_0+\sigma_{ave}\Delta t}\sum_{k=1}^{P}A_{z,k}\Big|_1^{n}\end{aligned} \tag{11}$$

3. Validation of the Proposed Models

A.. Pulse Reflection From a Coated Ideal Conductor

We start with a problem of a TE-polarized pulse reflecting from a metal wall coated with a dispersive layer. The parameters are of the Lorentz type. The permittivity has one resonance, while the permeability has two resonances. The parameters for the permeability are $\mu_\infty=1$, $\beta_{m,1}=4\cdot10^{20}\left(\mathrm{rad/s}\right)^2$, $\beta_{m,2}=1.25\cdot10^{21}\left(\mathrm{rad/s}\right)^2$, $\omega_{pm,1}=2\cdot10^{10}\left(\mathrm{rad/s}\right)$, $\omega_{pm,2}=5\cdot10^{10}\left(\mathrm{rad/s}\right)$, $\gamma_{m,1}=\gamma_{m,2}=1$, $\delta_{m,1}=5\cdot10^{9}\left(\mathrm{rad/s}\right)$

and $\delta_{m,2} = 5 \cdot 10^9 (\text{rad/s})$. The permittivity has parameters $\varepsilon_\infty = 2$, $\beta_{e,1} = 9 \cdot 10^{20} (\text{rad/s})^2$, $\omega_{pe,1} = 3 \cdot 10^{10} (\text{rad/s})$, $\gamma_{e,1} = 1$, and $\delta_{e,1} = 5 \cdot 10^8 (\text{rad/s})$. The electrical conductivity is taken to be zero.

The goal here is to demonstrate how the thickness of the coating affects the results. The material parameters above were used to calculate the results in Fig.4-6. The agreement with the results from the auxiliary variable method in [9] is seen to be good in the time domain. It is seen that the oscillations of the total wave become smaller when the thickness of the coating is decreased. This is natural since a PEC wall is obtained in the limit $d = 0$.

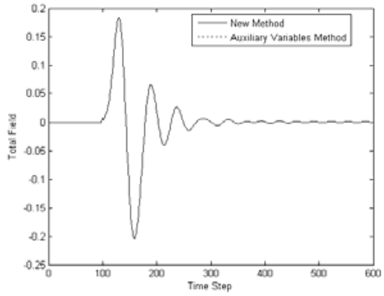


Fig. 4. d=1.2mm

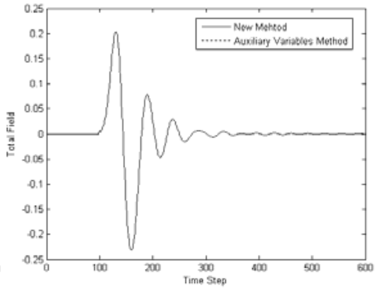


Fig. 5. d=1.5mm

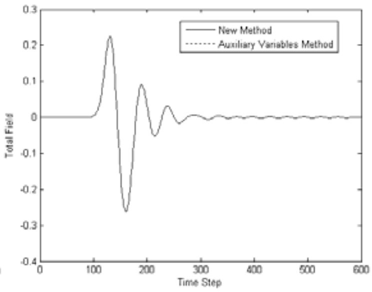
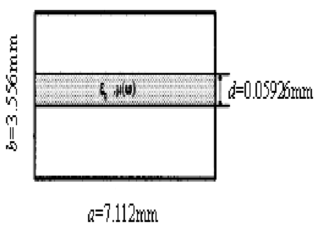


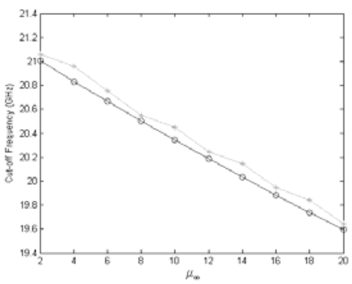
Fig. 6. d=1.8mm

B. Cutoff Frequency of a Loaded Waveguiede

Consider a rectangular waveguide with the widths of the walls equal to $a = 7.112\text{mm}$ and $b = 3.556\text{mm}$ as shown in Fig. 7. Suppose there is a thin magnetic layer of thickness d along the broader wall in the middle of the waveguide. The permeability of the layer is taken to be of the Lorentz type with a single pole pair. We keep the thickness of the layer fixed and present detailed validation of the model by varying the material parameters and by comparing with the exact results.

The cutoff frequency of the TE10 mode is obtained by requiring that $k_x = 0$ and solving for ω [11]. We first take the permeability to be independent of the frequency and vary the relative permeability. The agreement with the analytical results is rather good: the maximum relative error in Fig. 8 is approximately 1%. Then we fix $\mu_\infty = 1$, $\omega_{0,m} = 10 \cdot 10^{10} (\text{rad/s})$, and vary $\beta_m = \omega_{pm}^2$. The results are shown in Fig. 9 and maximum relative error is less than 1%.





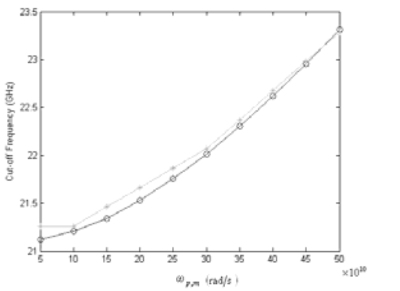


Fig. 7. Cross section of a rectangular waveguide Fig. 8. Cutoff frequencies versus μ_∞ Fig.9 Cutoff frequencies versus $\omega_{p,m}$

In Fig.10, $\omega_{0,m} = 15 \cdot 10^{10} (\text{rad/s})$, being larger than the cutoff of an empty waveguide ($\omega_{c,TE10} = \pi \cdot 4.22 \cdot 10^{10} (\text{rad/s})$). The result is seen to be clearly different from that in Fig.9. Finally, we vary the resonant frequency of the layer around the cutoff frequency of the unloaded waveguide. We set $\mu_\infty = 1$ and $\omega_{p,m} = \pi \cdot 4.22 \cdot 10^{10} (\text{rad/s})$. As shown in Fig. 11, the cutoff frequency of the loaded waveguide is seen to converge toward the cutoff frequency of the empty waveguide when we move away from the resonant frequency of the layer.

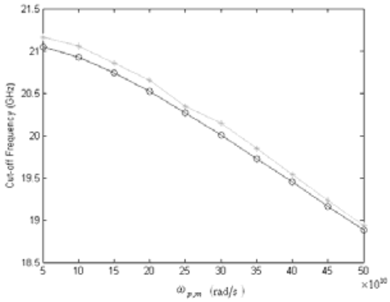


Fig. 10 Cutoff frequencies versus $\omega_{p,m}$

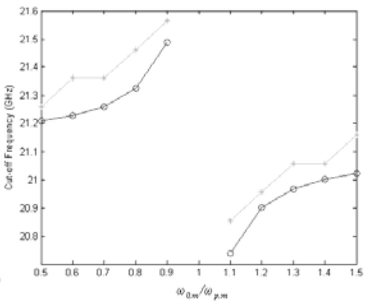


Fig. 11 Cutoff frequencies versus $\omega_{0,m}$

4. Conclusion

A new model for treating electrically thin dispersive layers and coatings in FDTD simulations was introduced. The most important feature of the model is its ability to accurately model dispersive layers having multiple resonances of material parameters. Then, the model is used for pulse reflection from a coated ideal conductor and waveguides loaded with a dispersive layer. The proposed models were numerically verified with the results from auxiliary variables method and analytic method.

Acknowledgements

This work was supported by the National Natural Science Fund (No: 50807003), the State Key Laboratory of Millimeter Waves (No: K201012).

References

[1] K. S. Yee, “Numerical solution of initial boundary value problems involving Maxwell’s equations in isotropic media”, IEEE Transaction On Antennas and Propagation, Vol. 14, No.3, pp. 302-307, 1966.

[2] A. Taflove, Susan C. Hagness, Computation Electrodynamics: The Finite-Difference Time-Domain Method. (Second Edition) Boston·London: Artech House, 2000.

[3] C. F. Lee, R. T. Shin, and J. A. Kong, “Time domain modeling of impedance boundary conditions,” IEEE Trans. Microwave Theory Tech., vol. 40, pp. 1847–1850, Sept. 1992.

[4] B. Z. Wang, “Time-domain modeling of the impedance boundary condition for an oblique incident parallel-polarization plane wave,” Microwave Opt. Technol. Lett., vol. 7, pp. 19–22, 1994.

[5] B. Z. Wang, “Time-domain modeling of the impedance boundary condition for an oblique incident perpendicular-polarization plane wave,” Microwave Opt. Technol. Lett., vol. 7, pp. 355–359, 1994.

[6] C. W. Penney, R. J. Luebbers, and J. W. Schuster, “Scattering from coated targets using a frequency-dependent, surface impedance boundary condition in FDTD,” IEEE Trans. Antennas Propagat., vol.44, pp. 434–443, Apr. 1996.

[7]. J. G. Maloney and G. S. Smith, “A comparison of methods for modeling electrically thin dielectric and conducting sheets in the finite-difference time-domain method,” IEEE Trans. Antennas Propagat., vol. 41, pp. 690–694, May 1993.

[8] J. L. Young, “Propagation in linear dispersive media: Finite-difference time-domain methodologies,” IEEE Trans. Antennas Propagat., vol. 43, pp. 422–426, Apr. 1995.

[9] M. K . Karkkainen, “Subcell FDTD modeling of electrically thin dispersive layers,” IEEE Trans. Antennas Propagat., vol. 51, pp. 1774–1780, June 2003.

[10] D. M. Sullivan, Electromagnetic simulation using the FDTD method, IEEE Press Series on RF and Microwave Technology, New York, 2000.

[11] S. A. Tretyakov, A. S. Cherepanov, and M. I. Oksanen, “Averaging method for analysing waveguides with anisotropic filling,” Radio Sci., vol. 26, no. 2, pp. 523–528, 1991.